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Process Identification of Open-Loop Unstable Systems

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Frequency response information plays a vital role in the design of process control systems. This information can be developed from a dynamic mathematical model or from experimental tests on the plant itself. Among the experimental techniques, step testing and pulse testing are the most widely used for dynamic identification. A review of literature revealed many applications of the pulse and step testing method for identification of the open-loop stable processes (for example, see Clements and Schnelle 1963, Hougen 1964, Schork and Deshpande 1978). But there appears to be no published information on experimental methods for identifying open-loop unstable processes. Instead, investigators have relied on dynamic mathematical models to obtain the open-loop transfer functions (for example, see Hopkins 1976). Of course, this is understandable, since experimental open-loop tests on such processes are not desirable.

An important example of an open-loop unstable system is an exothermic chemical reactor. In this case, a decreased heat transfer rate, accomplished by changing the rate of coolant flow, increases reactor temperature at which the reaction rate is higher which further increases the temperature. Thus, the system is open-loop unstable. The system can be stabilized by providing sufficient feedback by means of a controller. To determine suitable tuning constants for this controller, the open-loop transfer functions must be available. These tuning constants can be obtained if a dynamic model of the open-loop system is available. But in the absence of kinetic data, theoretical deveplopment of the model is not feasible.

The object of this work is to describe a novel technique for dynamic identification of open-loop unstable processes. The technique has been applied to a commercial exothermic chemical reactor which is used in the manufacture of a polymer. The method yields an approximate transfer function of the open-loop process, and is helpful in developing suitable tuning constants for the feedback controller.

THE METHOD

The block diagram of a typical exothermic reactor control system is shown in Figure 1. The reactor is assumed to have a transfer function arising from a first-order lag plus dead-time model. The form chosen for the transfer function is typical for many polymerization systems.

Our objective is to identify the open-loop transfer function and also the parameters of the reactor model. We assume that the dynamics of the measuring element and the final control element are known and can be expressed as

$$H(s) = \frac{k_T}{\tau_T s + 1} \tag{1}$$

where

k_T = transmitter gain, psi/0_c

τ_T = transmitter time constant, min.

and

$$G_v(s) = \frac{k_v}{\tau_v s + 1} \tag{2}$$

where

k_v = valve gain cal/hr/psi

τ_v = valve time constant, min.

The closed-loop transfer function of the system of Figure 1 to changes in set point is given by

$$\frac{T(s)}{R(s)} = \frac{G_c(s)\, G_v(s)\, G_p(s)}{1 + G_c(s)\, G_v(s)\, G_p(s)\, H(s)} \tag{3}$$

For simplicity Equation (3) is written as

$$\frac{T(s)}{R(s)} = \frac{G(s)}{1 + G(s)\, H(s)} \tag{4}$$

where $G(s)$ is the product of the transfer functions in the numerator of Equation (3).

The application of the present method requires a unity feedback on the closed-loop. Therefore, the block diagram of Figure

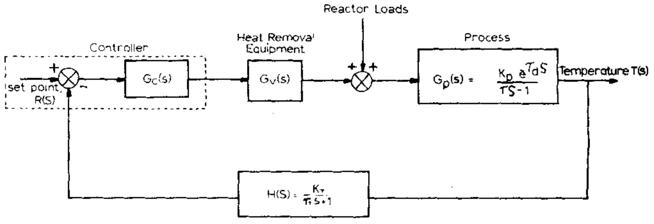

Figure 1. Block diagram of the reactor control system.

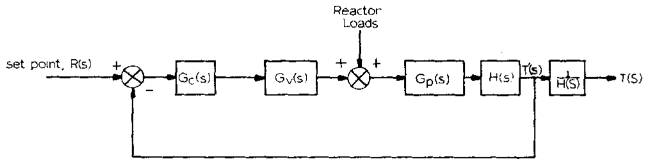

Figure 2. Block diagram of equivalent reactor control system having a unity feedback.

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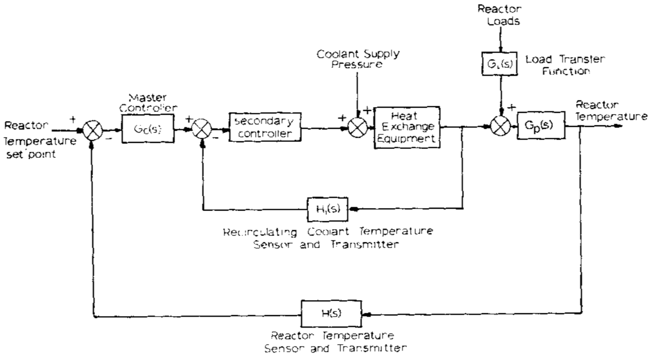

Figure 3. Cascade system for control of temperature.

1 is rearranged as shown in Figure 2. The closed loop transfer function relating $T'(s)$ to $R(s)$ is

$$\frac{T'(s)}{R(s)} = \frac{G(s)\,H(s)}{1 + G(s)\,H(s)} \tag{5}$$

and since

$$\frac{T(s)}{T'(s)} = \frac{1}{H(s)} \tag{6}$$

The relationship between $T'(s)/R(s)$ and $T(s)/R(s)$ is

$$\frac{T'(s)}{R(s)} = H(s)\,\frac{T(s)}{R(s)} \tag{7}$$

Equation (7) suggests that if the closed-loop frequency response diagram of $T(s)/R(s)$ is available, then the closed-loop frequency response diagram of $T'(s)/R(s)$ can be prepared according to Equation (7), provided $H(s)$ is known. Thus,

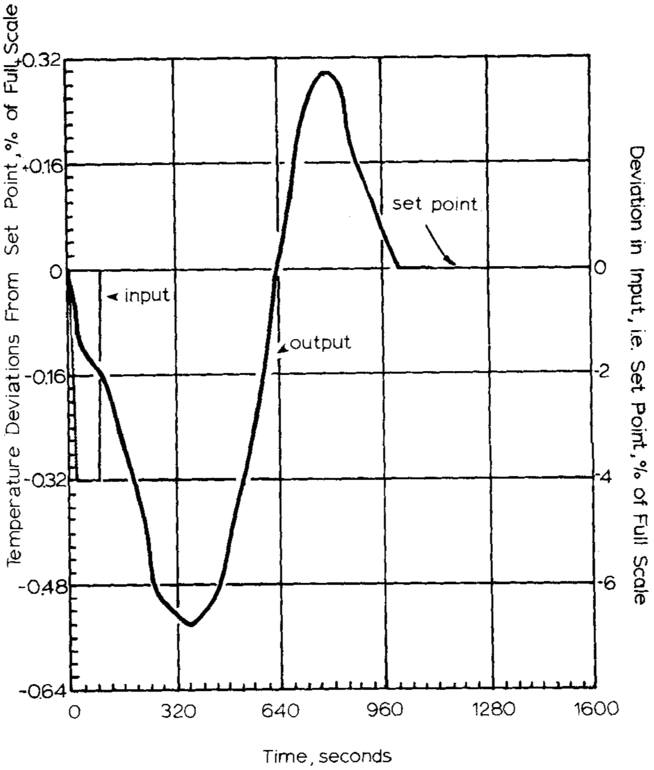

Figure 4. Input/output records from pulse test.

$$M = \left|\frac{T'}{R}(i\omega)\right| = \left|\frac{T}{R}(i\omega)\right|\,\left|H(i\omega)\right|$$

and (8)

$$\alpha = \measuredangle \frac{T'}{R}(i\omega) = \measuredangle \frac{T}{R}(i\omega) + \measuredangle H(i\omega)$$

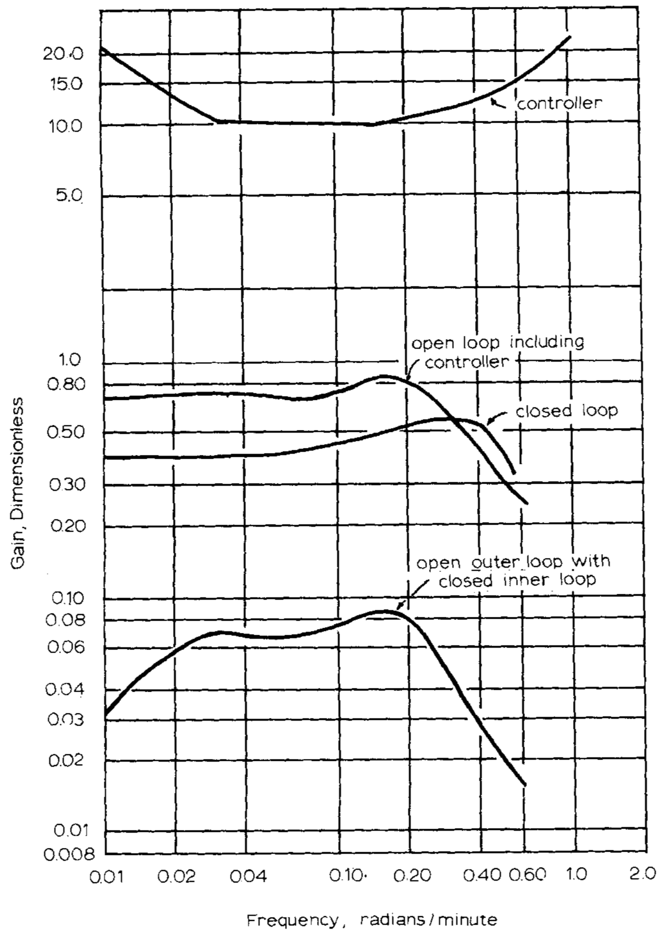

Figure 5A. Frequency response diagram—amplitude ratio plot.

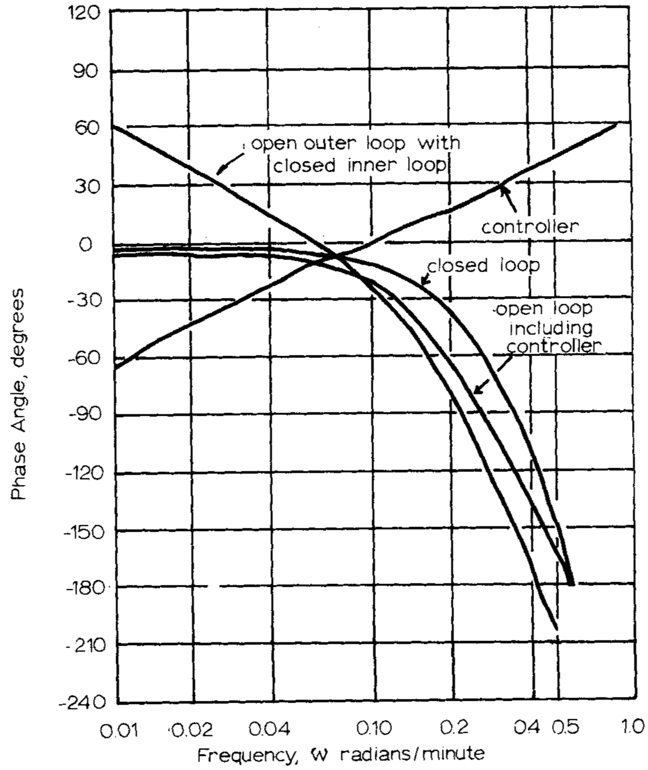

Figure 5B. Frequency response diagram—phase angle plot.

TABLE 1. TUNING CONSTANTS

	Proportional Band, %	Reset, Minutes	Derivative Time, Minutes
Secondary Controller	25	0.1	
Primary Controller			
Existing constants	10	50	2
Ziegler-Nichols settings	4	7.48	1.87
Luyben's settings	25	—not recommended—	

Each value of the closed-loop magnitude ratio, M and the phase angle, α corresponds to a *unique* value of the open-loop magnitude ratio A and the phase angle θ. Knowing M and α from the closed-loop tests, A and θ can be obtained from the Nichols chart (James et al., 1947). It may be noted that

$$A = |G_c(i\omega)|\,|G_v(i\omega)|\,|G_p(i\omega)|\,|H(i\omega)|$$

and (9)

$$\theta = \measuredangle G_c(i\omega) + \measuredangle G_v(i\omega) + \measuredangle G_p(i\omega) + \measuredangle H(i\omega)$$

The closed-loop tests are performed with a particular set of tuning constants implemented on the feedback controller so that $|G_c(i\omega)|$ and $\measuredangle G_c(i\omega)$ are available. Therefore, the only unknown is $G_p(i\omega)$, which can be obtained by graphical manipulations on the Bode' plot of A and θ.

Further, if only the open-loop, composite Bode' plot of the elements $G_v(s)\, G_p(s)\, H(s)$, is desired for controller tuning purposes, it can be easily obtained by taking away the contributions of the controller element, $G_c(i\omega)$, from the Bode' plot of A and θ. Then, the suitable tuning constants can be obtained by Luyben's method (Luyben and Melcic 1978).

The closed-loop frequency response diagram of $T/R(i\omega)$ is prepared from a pulse test. With the process operating in automatic (the tuning constants for the controller are determined by trial and error and are not necessarily the best ones), a pulse of predetermined magnitude and duration is introduced into the set point of the feedback controller (R in Figure 1) and the transient closed-loop response (T in Figure 1) as well as the input pulse are recorded. The input-output data are analyzed by Fourier transforms to generate the closed-loop frequency response diagram (Luyben 1973).

PLANT RESULTS AND DISCUSSION

In this particular plant installation, there are frequent disturbances in the supply pressure and temperature of the coolant as there are multiple users of the coolant which require varying amounts of cooling. Therefore, the control loop employed is a cascade loop instead of a simple feedback loop for control of temperature. The block diagram of the cascade control system is shown in Figure 3.

The cascade control system introduces additional elements into the control loop, and the application of the technique described in the previous section to extract a reactor model is somewhat more difficult. However, the inner loop is still potentially open-loop unstable, and our method can be applied to develop the open-loop frequency response diagram of the outer loop with the inner loop in automatic. The controller tuning constant for the inner loop were selected according to the guidelines of Shinskey (1967). The tuning constants for the Master Controller were essentially selected by the plant operator.

With both loops operating in automatic, a pulse of predetermined magnitude and duration was introduced into the set point of the master temperature controller, and the response of temperature was recorded. The input/output records of the pulse test are shown in Figure 4. The frequency response diagram of the closed-loop resulting from these records is shown in Figure 5. In this plant installation, the lag of the temperature transmitter ($H(s)$ in Figure 3) turned out to be negligible. Numerous values of the closed-loop amplitude ratios M's and phase angles α's are used in conjunction with the Nichols chart to obtain open loop magnitude ratios (note that this open-loop includes the closer inner loop) and phase angles. These include the contribution of the primary controller. The frequency response diagram of the open-loop system and the controller are also shown in Figure 5. Graphical subtraction of the contribution of the controller from the open-loop Bode plot gives the Bode plot of the open-loop system, with the closed inner loop, without the controller as shown in Figure 5.

The Ziegler-Nichols tuning constants (Ziegler and Nichols 1942) developed from Figure 5 and those recommended by Luyben (1978) are shown in Table 1. The proportional band which the operators had chosen lies between the Ziegler-Nichols value and the one recommended by Luben. The derivative times existing on the controller are approximately the same but the integral times are quite different. The integral time on the controller was reduced to 10 minutes as a result of these observations Subsequent experience has shown that the new tuning constants are adequate.

CONCLUSIONS

Presented here is a useful technique for process identification of potentially open-loop unstable systems. The method should be a valuable tool for the control system design of exothermic chemical reactors.

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NOTATION

A	= open-loop amplitude ratio
$G(s)$	= transfer function
$H(s)$	= transfer function of temperature transmitter
i	= $\sqrt{-1}$
k	= gain
M	= closed-loop amplitude ratio
P	= period, $2\pi/\omega_{co}$, time/cycle
R	= temperature set point
s	= laplace transform variable
T	= temperature
T'	= variable in equation 5

Greek Letters

ω	= frequency, radians/time
α	= closed-loop phase angle, degrees
θ	= open-loop phase angle, degrees
τ	= time constant, minutes

Subscripts

c	= pertaining to controller
co	= pertaining to cross-over frequency
d	= delay, seconds
I	= pertaining to controller
p	= pertaining to process
u	= pertaining to ultimate gain
v	= pertaining to valve

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An Empirical Model of Velocity Profiles for Turbulent Flow in Smooth Pipes

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The universal velocity profile developed by Nikuradse (1932) is used extensively to calculate velocity profiles. However, the velocity gradient predicted does not go to zero at the pipe centerline, and the integrated velocity profile and resulting friction factor equation do not agree with the universal law of friction of Nikuradse (1932).

Here, we suggest a new model profile, similar to that of Bogue and Metzner (1963). This model differs from others in that the parameters are adjusted so that the centerline velocity gradient is zero, and further, when the model is integrated, the universal law of friction is obtained.

A modified dimensionless velocity u^+ in the turbulent core may be expressed by adding the correction function $h(\eta)$ to the universal velocity profile to yield

$$u^+ = \frac{1}{K}\left[\ln y^+ + h(\eta)\right] + B \tag{1}$$

where y^+ is the local Reynolds number and η is the dimensionless radial distance from the pipe wall. The parameters K and B are constants. A definition of $h(\eta)$ suggested by Hinze (1975) is

$$h(\eta) = C \exp\left[-\frac{1}{2}\left(\frac{\eta - .8}{\sigma}\right)^2\right] \tag{2}$$

In this form Bogue and Metzner suggest $C = 0.05(2.0/f)^{1/2}$ and $\sigma^2 = 0.15/2.0$. But introducing the friction factor into Equation (2) will not allow the velocity gradient at $\eta = 1$ to be zero unless the constant K is replaced by a function of the Reynolds number. Dependence of K on Reynolds number was not found by Schlicting (1965) who tested the friction factor data of various investigators with the universal law of friction.

Here, two boundary conditions are used to evaluate as constants C and σ in Equation (2). First, requiring the centerline dimensionless velocity gradient to be zero implies

$$h'(1) = -\frac{0.2C}{\sigma^2}\exp\left[\frac{-0.02}{\sigma^2}\right] = -1 \tag{3}$$

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Solving for the parameter C

$$C = 5\sigma^2 \exp\left[\frac{0.02}{\sigma^2}\right] \tag{4}$$

A second condition comes from Ross's (1953) analysis of Nikuradse's velocity data. Ross plotted the velocity data as a straight line on a semi-logarithmic graph up to η equal to 0.15, where deviation from the universal velocity profile occurred, and extrapolated the curve until it attained the value of $u^+ = 1.0$, obtaining

$$u^+(\text{actual})|_{\eta=1} = u_p^+|_{\eta=1.38\pm0.04} = 1.0 \tag{5}$$

where

$$u_p^+ = \frac{1}{K}\ln(\eta Re^*) + B \tag{6}$$

Equation (6) represents a general form of the universal velocity profile. The product of η and Re^* equal y^+. Equating u^+ from Equation (6) evaluated at $\eta = 1.38 \pm 0.04$ with u^+ from Equation (1) evaluated at $\eta = 1$ gives

$$h(1) = \ln(1.38 \pm 0.04) \tag{7}$$

Substituting Equations (4) and (7) into Equation (2) at $\eta = 1$ and taking the square root yields

$$\sigma = 0.254 \tag{8}$$

Equation (4) is then used to find

$$C = 0.439 \tag{9}$$

The dimensionless velocity profile in the turbulent core is then

$$u^+ = \frac{1}{K}\left[\ln y^+ + .439 \exp\left[-\frac{(\eta - .8)^2}{.129}\right]\right] + B \tag{10}$$

Before the constants K and B can be found the quantity $(u^+|\eta = 1 - u^+)$ is area averaged over the pipe cross-section